Main Outlines

- Review of self inductance
- Concept of mutual inductance
- Mutual inductance in terms of self inductance

Terms & Definitions

- ✓ **Inductor-** A device that introduces inductance into an electrical circuit (usually a coil)
- ✓ **Inductance-** The property of an electric circuit when a varying current induces an EMF in that circuit or another circuit
- ✓ **Self-inductance-** The property of an electric circuit when an EMF is induced in that circuit by a change of current
- ✓ **Henry -** The unit of inductance
- ✓ **Permeability-** The measure of the ease with which material will pass lines of flux
- ✓ **Mutual Inductance-** The property of two circuits whereby an EMF is induced in one circuit by a change of current in the other

Flux Linkages and Faraday's Law

- **The flux linking a coil with N turns:** $\lambda = N\,\phi$

- **Faraday's law of magnetic induction:** $e = \dfrac{d\lambda}{dt}$

- The voltage induced in a coil whenever its flux linkages are changing.

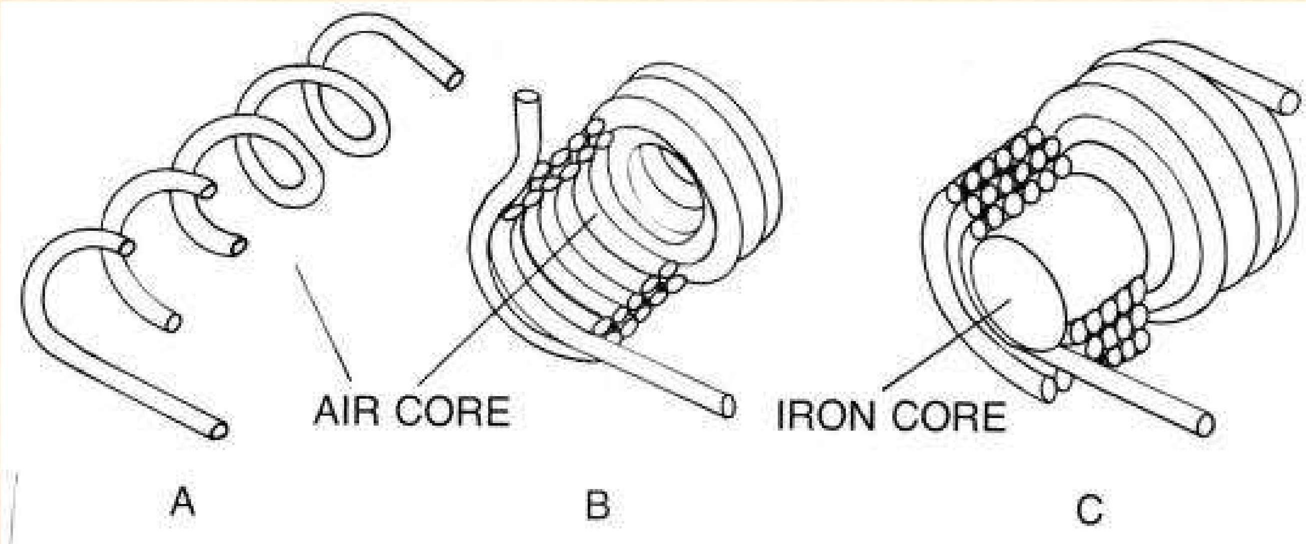
- Changes occur from:
  - Magnetic field changing in time
  - Coil moving relative to magnetic field

Lenz's Law

❑ **Lenz's law** states that the polarity of the induced voltage is such that the voltage would produce a current (through an external resistance) that opposes the original change in flux linkages.

➢ The current in a conductor, as a result of an induced voltage, is such that the magnetic flux due to it is **opposite to** the magnetic flux that caused the induced voltage

Types of Inductors



$$\Re = \frac{1}{P} = \frac{l}{\mu_0 \mu_r A}$$

$$L = \frac{N^2}{\Re} = N^2 P$$

Self Inductance

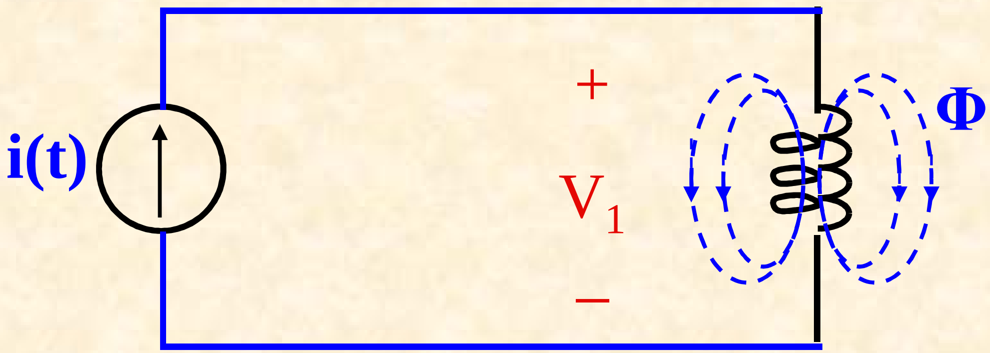
- It called **self inductance** because it relates the voltage induced in a coil by a time varying current in the same coil

- Consider a single inductor with **N** number of turns when current, ***i*** flows through the coil, a magnetic flux, **Φ** is produces around it

$$\phi = \frac{(N\,i)}{\Re} = (N\,i)\,P$$

$$\lambda = N\,\phi = N\,\frac{(N\,i)}{\Re}$$

$$\lambda = \left(\frac{N^2}{\Re}\right) i = \left(N^2 P\right) i$$



$$L = \frac{N^2}{\Re} = N^2\,P$$

$$\lambda = N\,\phi = L\;i$$

Self Inductance

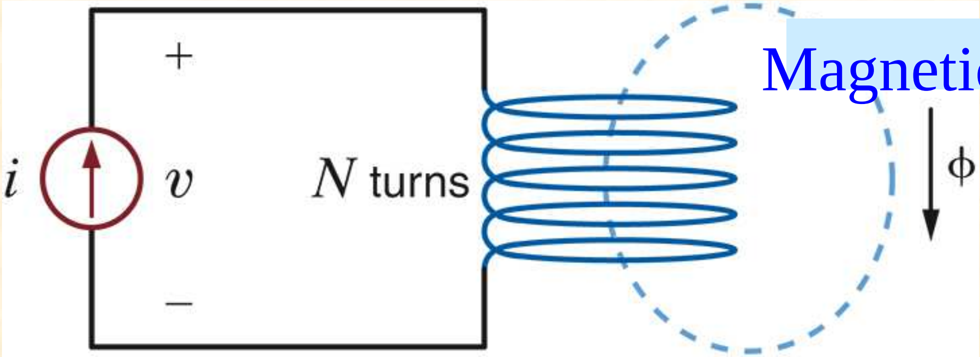
- According to Faraday's Law, the voltage, (v) induced in the coil is proportional to (N) number of turns and rate of change of the magnetic flux, Φ;

$$v = N\frac{d\phi}{dt}$$

- In addition, the induced voltage, (v) can be written in terms of the self inductance, (L) and rate of change of the current, (i);

$$v = L\frac{di}{dt}$$

Self Inductance (conclusions)



Magnetic field

$$\lambda = N\phi$$

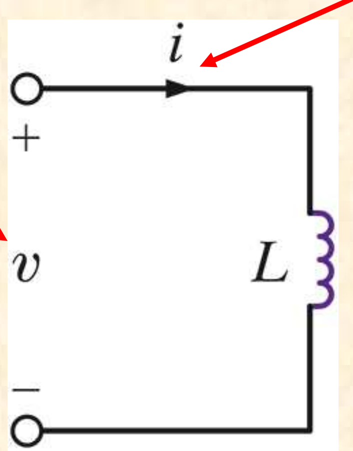
Total magnetic flux linked by N-turn coil

$$v = \frac{d\lambda}{dt}$$

Faraday's Induction Law

$$\lambda = Li$$

Ampere's Law



$$v = L\frac{di}{dt}$$

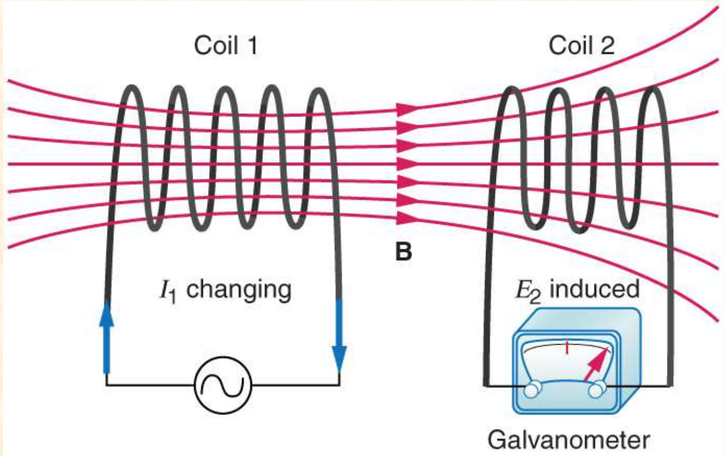
$$L = \frac{N^2}{\Re} = N^2\,P$$

Ideal Inductor

Mutual Inductance

- **Mutual inductance** occurs when a changing current in one circuit results, via changing magnetic flux, in an induced emf and thus a current in an adjacent circuit

  - The coils are said to have mutual inductance M, which can either add or subtract from the total inductance depending on if the fields are aiding or opposing



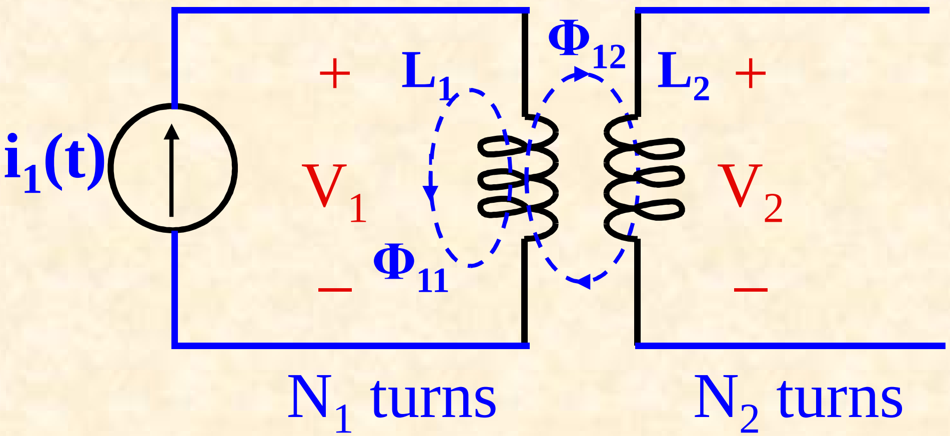
  - **Mutual inductance** is the ability of one inductor to induce a voltage across a neighboring inductor

Mutual Inductance

Consider the following two cases:

- **Case 1:**

two coil with self – inductances L_1 and L_2 which are in close proximity which each other. Coil 1 has N_1 turns, while coil 2 has N_2 turns



Mutual Inductance

- Magnetic flux $\mathbf{\Phi_1}$ from coil 1 has two components;

  * Φ_{11} links only coil 1

  * Φ_{12} links both coils

✓ Hence; $\Phi_1 = \Phi_{11} + \Phi_{12}$

where

$$\phi_1 = \frac{N_1 i_1}{\Re_1} = N_1 i_1 \; P_1$$

Total flux

Leakage flux

$$\phi_{11} = \frac{N_1 i_1}{\Re_{11}} = N_1 i_1 \; P_{11}$$

$$\phi_{12} = \frac{N_1 i_1}{\Re_{12}} = N_1 i_1 \; P_{12}$$

Magnetizing (Mutual) flux

$$P_1 = P_{11} + P_{21}$$

Mutual Inductance

- Thus; the voltage induces in coil 1

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$v_1 = N_1 \frac{d}{dt}(\phi_{11} + \phi_{12})$$

$$v_1 = N_1^2 (P_{11} + P_{12}) \frac{di_1}{dt}$$

$$v_1 = \left(N_1^2 P_1\right) \frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

Mutual Inductance

✓ The Voltage induces in coil 2

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

$$\phi_{12} = \frac{N_1 i_1}{\Re_{12}} = N_1 i_1 \, P_{12}$$

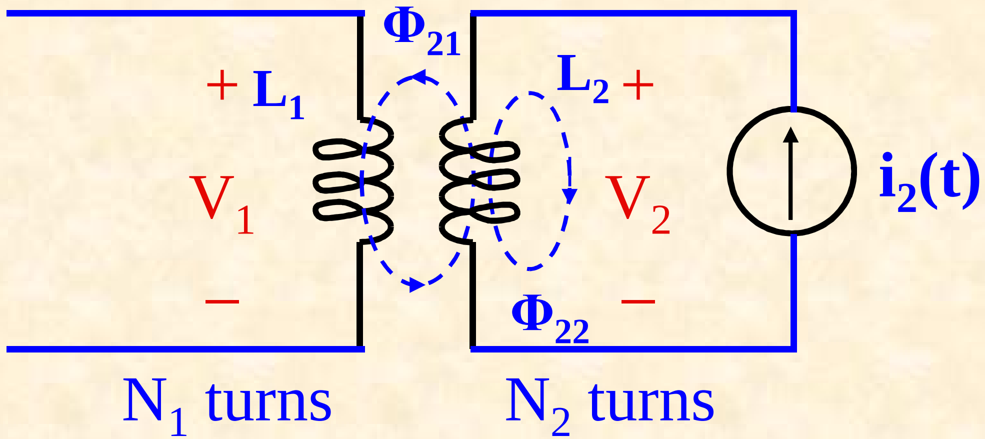
$$v_2 = N_2 N_1 \, P_{12} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

Subscript 21 in M_{21} means the mutual inductance on coil 2 due to coil 1

$$M_{21} = \frac{N_2 N_1}{\Re_{12}} = N_2 N_1 \, P_{12}$$

Mutual Inductance

- **Case 2:** Same circuit but let current i_2 flow in coil 2.



✓ The magnetic flux $\mathbf{\Phi_2}$ from coil 2 has two components:

*** Φ_{22} links only coil 2**

*** Φ_{21} links both coils**

Mutual Inductance

- Hence; $\Phi_2 = \Phi_{22} + \Phi_{21}$

where

Leakage flux

$$\phi_{22} = \frac{N_2 i_2}{\Re_{22}} = N_2 i_2 \; P_{22}$$

$$\phi_2 = \frac{N_2 i_2}{\Re_2} = N_2 i_2 \; P_2$$

Total flux

$$\phi_{21} = \frac{N_2 i_2}{\Re_{21}} = N_2 i_2 \; P_{21}$$

Magnetizing (Mutual) flux

$$P_2 = P_{22} + P_{12}$$

Mutual Inductance

- ✓ Thus; the voltage induced in coil 2

$$v_2 = N_2 \frac{d\phi_2}{dt}$$

$$v_2 = N_2 \frac{d}{dt}(\phi_{22} + \phi_{21})$$

$$v_2 = N_2^2 (P_{22} + P_{21}) \frac{di_2}{dt}$$

$$v_2 = \left(N_2^2 P_2\right) \frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

Mutual Inductance

✓ The Voltage induces in coil 1

$$v_1 = N_1 \frac{d\phi_{21}}{dt}$$

$$\phi_{21} = \frac{N_2 i_2}{\Re_{21}} = N_2 i_2 \, P_{21}$$

$$v_1 = N_1 N_2 \, P_{21} \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

Subscript 12 in M_{12} means the mutual inductance on coil 1 due to coil 2

$$M_{12} = \frac{N_1 N_2}{\Re_{21}} = N_1 N_2 \, P_{21}$$

Mutual Inductance

- Since the two circuits and two current are the same:

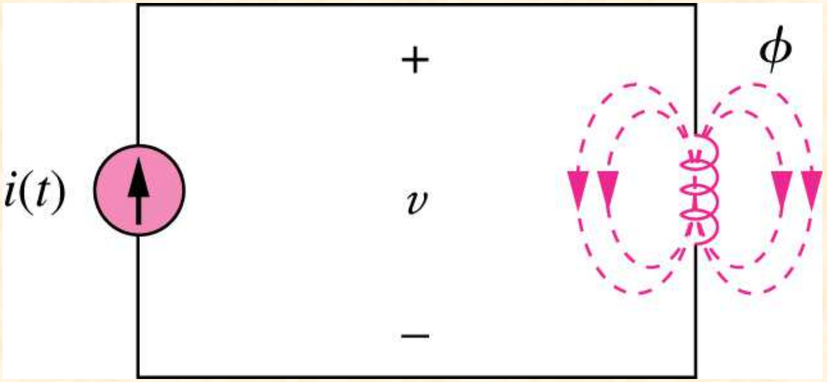
For a linear system

$$\Re_{21} = \Re_{12}$$

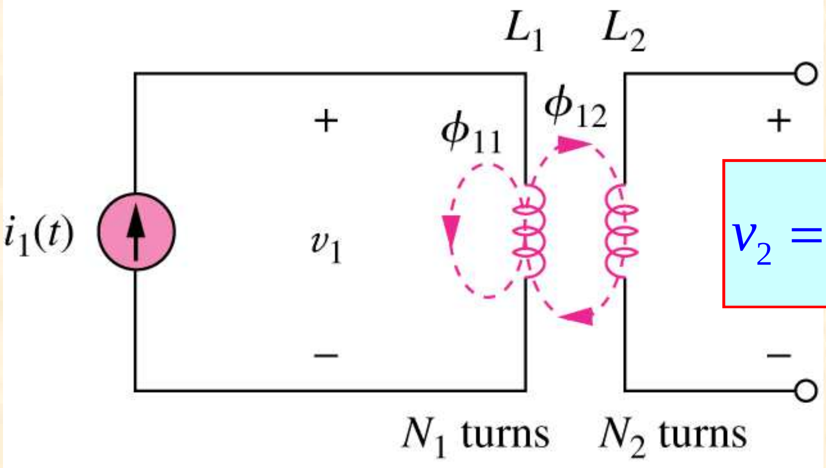
$$\boxed{M_{21} = M_{12} = M}$$

- Mutual inductance **M** is measured in Henrys (H)

Mutual Inductance (conclusions)



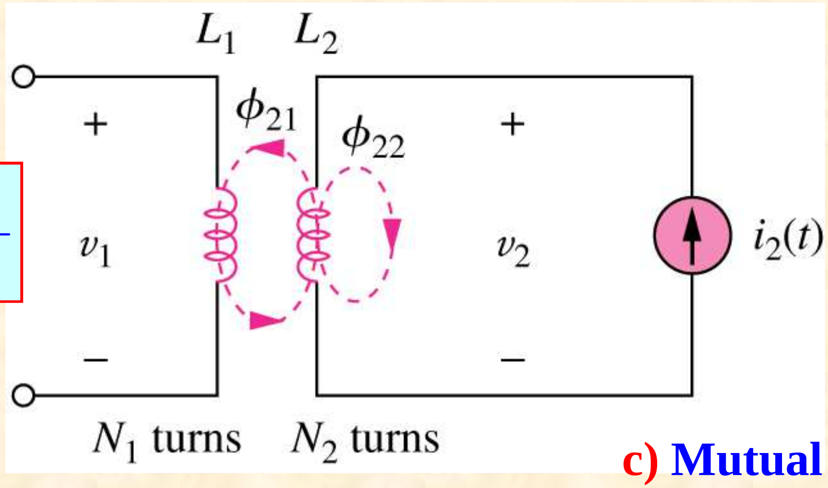
a) Magnetic flux produced by a single coil



$$v_2 = M_{21}\frac{di_1}{dt}$$

b) Mutual inductance M_{21} of coil 2 with respect to coil 1

$$v_1 = M_{12}\frac{di_2}{dt}$$



c) Mutual inductance of M_{12} of coil 1 with respect to coil 2

Mutual inductance in terms of self inductances

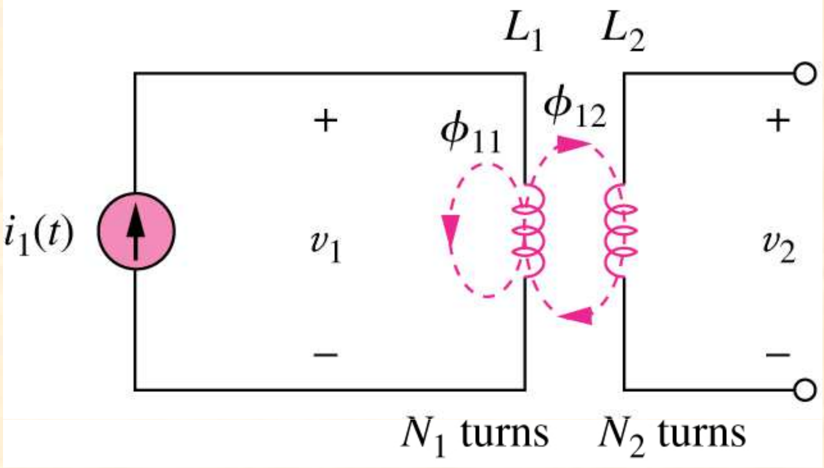
$$L_1 = N_1^2\,P_1$$

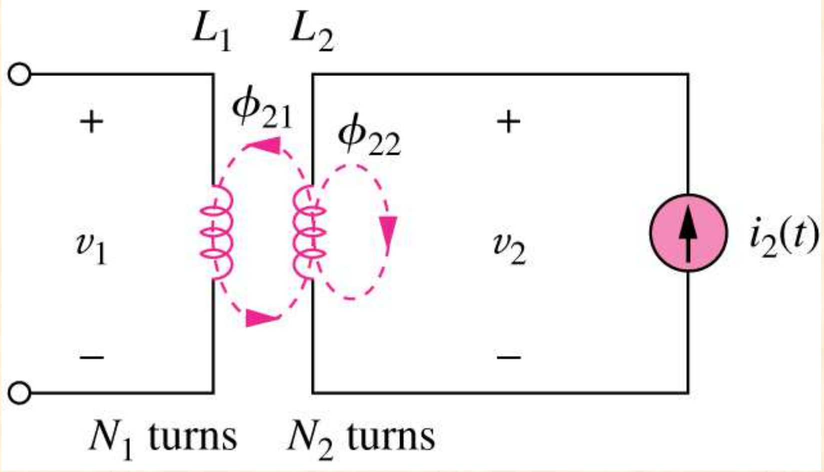
$$L_2 = N_2^2\,P_2$$

$$L_1 L_2 = N_1^2 N_2^2\,P_1 P_2$$

$$P_1 = P_{11} + P_{21}$$

$$P_2 = P_{22} + P_{12}$$





Mutual inductance in terms of self inductances

$$L_1 L_2 = N_1^2 N_2^2 \left(P_{11} + P_{21}\right)\left(P_{22} + P_{12}\right)$$

For a linear system, $P_{12} = P_{21}$

$$L_1 L_2 = N_1^2 N_2^2 \; P_{12}^2 \left(1 + \frac{P_{11}}{P_{12}}\right)\left(1 + \frac{P_{22}}{P_{12}}\right)$$

Mutual inductance in terms of self inductances

$$L_1 L_2 = (N_1 N_2 P_{12})^2 \left(1+\frac{P_{11}}{P_{12}}\right)\left(1+\frac{P_{22}}{P_{12}}\right)$$

$$\left.\begin{aligned} L_1 L_2 &= M^2 \left(1+\frac{P_{11}}{P_{12}}\right)\left(1+\frac{P_{22}}{P_{12}}\right) \\ \frac{1}{k^2} &= \left(1+\frac{P_{11}}{P_{12}}\right)\left(1+\frac{P_{22}}{P_{12}}\right) \end{aligned}\right\} \quad M^2 = k^2 L_1 L_2$$

Mutual inductance in terms of self inductances

➢ The mutual inductance can be written in terms of self inductances as:

$$M = k\sqrt{L_1 L_2}$$

✓ The constant **"k"** is called the coupling coefficient

$$\frac{1}{k^2} = \left(1 + \frac{P_{11}}{P_{12}}\right)\left(1 + \frac{P_{22}}{P_{12}}\right)$$ ➡ **Must be greater than 1**

✓ Therefore k ➡ **Must be less than 1**

Coupling Coefficient

- The coupling coefficient "**k**" is a measure of the percentage of flux from one coil that links another coil (a measure of the magnetic coupling between two coils). The coupling coefficient for 2 mutual inductors is given by:

$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

- The coupling coefficient "**k**" depends on the closeness of two coils, their core, their orientation and their winding

Coupling Coefficient

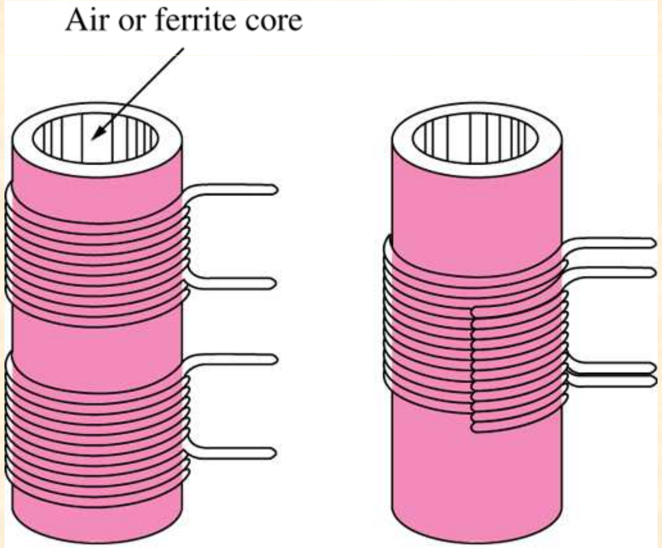
➢If **$k > 0.5$** , then most of the flux from the one coil links the other and the coils are said to be **tightly coupled**

➢ If **$k < 0.5$**, then most of the flux is not shared between the 2 coils and in this case the coils are said to be **loosely coupled**

❑ **Range of k : $0 \le k \le 1$**

✓ **$k = 0$** means the two coils are **not coupled**

✓ **$k = 1$** means the two coils are **perfectly coupled**



Loosely coupled coil **Tightly coupled coil**

Coupling Coefficient

k can be expressed in terms of flux as

$$k = \frac{\phi_{12}}{\phi_{11} + \phi_{12}}$$

or $$k = \frac{\phi_{21}}{\phi_{21} + \phi_{22}}$$

$k = 1$ means perfect coupling.

$$\Rightarrow \phi_{11} = \phi_{22} = 0$$